Project Scheduling for Maximum NPV with Variable Activity Durations and Uncertain Activity Outcomes

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Introduction:

Activity failure

- Common to many R&D-projects (especially NPD), but also occurs in other sectors: pharmaceuticals, chemicals, construction industry, software development, innovation, ...

- Individual activity failure results in overall project failure
  => project pay-off is not obtained
  - FDA review
  - toxicology tests
  - undesirable side effects
  - building permit
  - loan requests
  - market potential
  - patent infringement
  - ...

Problem Description:

Example
Problem description:

*Example: deterministic durations*
Problem description:
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Problem description: 
*Example: deterministic durations*

*Feasible schedule*

- **1**: -3M$ at time 1
- **2**: -1M$ at time 2
- **3**: -2M$ at time 3
- **4**: -12M$ at time 4
- **5**: -17M$ at time 5

Discounting: $c_i \times e^{-rt}$

Result = 39.60 M$
Problem description:

Example: deterministic durations

Feasible schedule
Problem description:

*Example: deterministic durations*
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*Example: deterministic durations*
Problem Description:

**Definitions**

- Stochastic activity durations (exponentially distributed) => use of a Continuous Time Markov Decision Chain
- Expected-NPV-objective: incurred cash flow $c_i$ at the start of activity $i$
- Optimization over the set of policies that start activities at the end of other activities
- Number of activities $n$
- Mean duration $d_i$ of activity $i$
- Activity $i$ has probability of technical success $p_i$
- Discount rate $r$
- No renewable resource constraints
Model Description:

*Stochastic durations – Continuous time Markov decision chain*

- **Preliminary concepts:**
  - Status of activity $i$ at time $t$:
    - Not started $\Omega_i(t)=0$
    - Started/in progress $\Omega_i(t)=1$
    - Started $\Omega_i(t)=2$
  - $\Omega(t)=(\Omega_0(t), \Omega_1(t), ..., \Omega_n(t))$ defines the state of the system

Size of statespace $Q$ has upper bound $|Q|=3^n$

Most of these states do not satisfy precedence constraints

$\Rightarrow$ a strict and clear definition of the statespace is essential

$\Rightarrow$ use of UDC-concept to define the statespace
Model Description:

UDC: max set of activities that can be executed in parallel
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Model Description:

Network of UDCs
Model Description:

*Illustration of statespace and backward SDP-recursion*
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States assigned to UDC:

\[(2,2,2,2,2,0) \rightarrow 80 \text{M}\$\]
Model Description:
*Illustration of statespace and backward SDP-recursion*
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States assigned to UDC: 
(2,2,2,2,1,0) 
(2,2,2,2,2,0) [80.00 M$]
Model Description:

*Illustration of state space and backward SDP-recursion*

States assigned to UDC:
\( (2,2,2,2,1,0) \)
\( (2,2,2,2,2,0) \) [80.00 M$]

Discount factor: \( \left(\frac{1}{d_i}\right) \times (r + \left(\frac{1}{d_i}\right))^{-1} \)
\( d_5 = 3 \Rightarrow \) Discount factor = 0.97
Discounted value at state entry = 77.67 M$
\( p_5 = 0.75 \Rightarrow \) NPV at state entry = 58.25 M$
Model Description:
Illustration of statespace and backward SDP-recursion

States assigned to UDC:
\[(2,2,2,2,2,1,0) \rightarrow 58.25\text{M}\$
\[(2,2,2,2,2,0) \quad [80.00\text{M}]\$

Discount factor : \((1/d_t)\times(r+(1/d_t))^{-1}\)
\[d_5 = 3 \Rightarrow \text{Discount factor} = 0.97\]
Discounted value at state entry = 77.67\text{M}\$
\[p_5 = 0.75 \Rightarrow \text{NPV at state entry} = 58.25\text{M}\$
Model Description:
Illustration of statespace and backward SDP-recursion

States assigned to UDC:
(2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,0,0)
Model Description:
*Illustration of statespace and backward SDP-recursion*

States assigned to UDC:

\[(2,2,2,2,2,1,0) \rightarrow 58.25M\$$
\[(2,2,2,2,2,0,0)\]

\[(2,2,2,2,2,1,0) \rightarrow [58.25M]$$
Model Description:
*Illustration of statespace and backward SDP-recursion*

States assigned to UDC:

(2,2,2,2,2,1,0) -> $58.25M$
(2,2,2,2,2,0,0)
(2,2,2,2,2,1,0) [58.25M$]

Only decision left is to start activity 5
=> incur cost $c_5 = -17M$
=> NPV at state entry = 41.25M$
Model Description:
Illustration of statespace and backward SDP-recursion

States assigned to UDC:
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,2,1,0) [58.25M$]

Only decision left is to start activity 5
=> incur cost $c_5 = -17M$
n=> NPV at state entry = 41.25M$
Model Description:
*Illustration of statespace and backward SDP-recursion*

States assigned to UDC:

- \((2,2,2,2,1,0) \rightarrow 58.25\text{M}\$\)
- \((2,2,2,2,0,0) \rightarrow 41.25\text{M}\$\)
- \((2,2,2,1,2,0)\)
Model Description:

*Illustration of statespace and backward SDP-recursion*

States assigned to UDC:

- $(2,2,2,2,2,1,0) \rightarrow 58.25\text{M}$
- $(2,2,2,2,2,0,0) \rightarrow 41.25\text{M}$
- $(2,2,2,2,1,2,0) \rightarrow 41.25\text{M}$
- $(2,2,2,2,2,2,0) \rightarrow 80.00\text{M}$
Model Description:
*Illustration of statespace and backward SDP-recursion*

States assigned to UDC:
- \((2,2,2,2,2,1,0) \rightarrow 58.25M\$\)
- \((2,2,2,2,2,0,0) \rightarrow 41.25M\$\)
- \((2,2,2,2,1,2,0) \rightarrow 76.92M\$\)
- \((2,2,2,2,2,2,0) \rightarrow 80.00M\$\)
Model Description: 
*Illustration of statespace and backward SDP-recursion*

States assigned to UDC:
- \((2,2,2,2,2,1,0) \rightarrow 58.25 \text{M\$}\
- \((2,2,2,2,2,0,0) \rightarrow 41.25 \text{M\$}\
- \((2,2,2,2,1,2,0) \rightarrow 76.92 \text{M\$}\
- \((2,2,2,2,1,1,0) \rightarrow \)
Model Description:
Illustration of statespace and backward SDP-recursion

States assigned to UDC:
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0) -> 76.92M$
(2,2,2,2,1,1,0)
(2,2,2,2,1,0) [58.25M$
(2,2,2,2,1,2,0) [76.92M$]
Model Description:
Illustration of statespace and backward SDP-recursion

States assigned to UDC:
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0) -> 76.92M$
(2,2,2,2,1,1,0)

(2,2,2,2,2,1,0) [58.25M$]
(2,2,2,2,1,2,0) [76.92M$]

Probability activity finishing first
=> (1/di) x (SUM(1/di)^-1)
Model Description:
Illustration of statespace and backward SDP-recursion

States assigned to UDC:
- $(2,2,2,2,1,0) \rightarrow 58.25M$
- $(2,2,2,2,0,0) \rightarrow 41.25M$
- $(2,2,2,1,2,0) \rightarrow 76.92M$
- $(2,2,2,1,1,0)$

43% $(2,2,2,2,1,0) \rightarrow [58.25M]$
57% $(2,2,2,2,1,2,0) \rightarrow [76.92M]$

Probability activity finishing first
$=> (1/d_i) \times (\text{SUM}(1/d_i))^{-1}$
Model Description:

Illustration of statespace and backward SDP-recursion

States assigned to UDC:

- \((2,2,2,2,1,0) \rightarrow 58.25M\$\)
- \((2,2,2,2,0,0) \rightarrow 41.25M\$\)
- \((2,2,2,1,2,0) \rightarrow 76.92M\$\)
- \((2,2,2,1,1,0) \rightarrow 56.96M\$\)

43% \((2,2,2,2,1,0) [58.25M]\$
57% \((2,2,2,2,1,2,0) [76.92M]\$

Probability activity finishing first

\[ => (1/d_i) \cdot (\text{SUM}(1/d_i))^{-1} \]
Model Description:
Illustration of statespace and backward SDP-recursion
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*Illustration of statespace and backward SDP-recursion*

States assigned to UDC:

- $(2,2,2,0,0,0,0)$
- $(2,2,2,1,0,0,0)$ $[21.51M\$]$
- $(2,2,2,0,1,0,0)$ $[17.46M\$]$
- $(2,2,2,0,0,1,0)$ $[18.26M\$]$
- $(2,2,2,1,1,0,0)$ $[17.46M\$]$
- $(2,2,2,1,0,1,0)$ $[18.26M\$]$
- $(2,2,2,0,1,1,0)$ $[14.26M\$]$
- $(2,2,2,1,1,1,0)$ $[14.17M\$]$

Not started
- Started
- Finished
Model Description:

*Illustration of statespace and backward SDP-recursion*
Model Description:

*Illustration of the optimal policy*
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Model Description:

_Illustration of the optimal policy_
Model Description:

Illustration of the optimal policy
## Computational performance (seconds):

**AMD Athlon (1.8GHz) – 2048MB RAM**

<table>
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<th>$n$</th>
<th>Number of networks analyzed</th>
<th>Average CPU-time</th>
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</table>

**OS:** Order strength; a measure of network density
Conclusions:

Contribution & future research

- **Contribution:** we develop a model that incorporates:
  - Stochastic activity durations
  - NPV-objective
  - Activity failure
  - Good computational performance (networks of 120 activities are solved to optimality)

- **Future research:**
  - Modular projects
  - General durations using Phase-Type distributions
  - Resources
  - Activity delay