Minimizing the makespan of a project with stochastic activity durations under resource constraints

Stefan Creemers

(December, 2014)

Agenda

- Problem setting:
  - Past work
  - The SRCPSP
  - Phase Type (PH) distributions
- Model discussion & comparison
- Results:
  - Solution quality
  - Computational performance
- Contribution

Problem setting

Creemers, Leus, Lambrecht (2010). Scheduling Markovian PERT networks to maximize the net present value, Operations Research Letters, 38, pp. 51-56.

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2. No resources
3. Exponentially-distributed activity durations
4. Use of a SDP recursion to obtain the optimal policy

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2. Renewable resources
3. General activity durations (PH approximation)
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4. **Use of an improved/modified SDP recursion**

Improvement of the SDP recursion

n	OS	% Solved	Average CPU (2010)	Average CPU (improved)	Average Factor
10	0.8	100%	0.00	0.00	**-**
10	0.6	100%	0.00	0.00	**-**
10	0.4	100%	0.00	0.00	**6.81**
20	0.8	100%	0.00	0.00	**-**
20	0.6	100%	0.01	0.00	**27.25**
20	0.4	100%	0.46	0.03	**17.60**
30	0.8	100%	0.01	0.00	**17.53**
30	0.6	100%	0.33	0.02	**14.90**
30	0.4	100%	26.92	1.49	**18.12**
40	0.8	100%	0.03	0.00	**12.41**
40	0.6	100%	6.62	0.49	**13.62**
40	0.4	97%	2,337.96	72.25	**32.36**
50	0.8	100%	0.15	0.01	**10.60**
50	0.6	100%	100.28	4.43	**22.62**
50	0.4	13%	52,267.30	823.71	**63.45**
60	0.8	100%	0.74	0.06	**12.36**
60	0.6	100%	2,210.08	67.87	**32.56**
60	0.4	0%	-	-	**-**

n	OS	% Solved	Average CPU (2010)	Average CPU (improved)	Average Factor
70	0.8	100%	3.19	0.24	**13.09**
70	0.6	73%	17,495.49	378.64	**46.21**
70	0.4	0%	-	-	**-**
80	0.8	100%	10.81	0.79	**13.65**
80	0.6	30%	72,473.41	1,188.01	**61.00**
80	0.4	0%	-	-	**-**
90	0.8	100%	50.64	3.15	**16.06**
90	0.6	0%	-	-	**-**
90	0.4	0%	-	-	**-**
100	0.8	100%	171.42	9.60	**17.85**
100	0.6	0%	-	-	**-**
100	0.4	0%	-	-	**-**
110	0.8	100%	1,193.88	40.93	**29.17**
110	0.6	0%	-	-	**-**
110	0.4	0%	-	-	**-**
120	0.8	100%	12,789.06	260.66	**49.06**
120	0.6	0%	-	-	**-**
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Improvement of the SDP recursion

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20	0.8	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	**61.00**
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	**16.06**
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	**17.85**
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
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In comparison with the model of Creemers et al. (2010), the computation speed has been increased by factor 56 (= **56 times faster**).

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	61.00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16.06
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-
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In comparison with the model of Creemers et al. (2010), the computation speed has been increased by factor 56 (= **56 times faster**).

When compared to the model of Sobel et al. (2009), the new model is even **840 times faster**.

Problem setting

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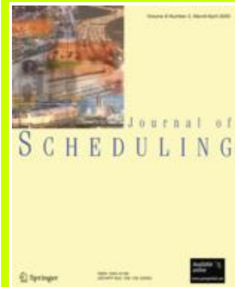

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Model extensions: PH distributions

- Introduced by Neuts in 1981
- A Phase Type (PH) distribution is a mixture of exponential distributions
- The exponential, Erlang, Coxian, and hyper-exponential distribution are all examples of a PH distribution
- We use simple PH distributions to match the first two moments of the distribution of the activity duration (more advanced PH distributions, however, can also be used)

PH distributions: Example of a single activity

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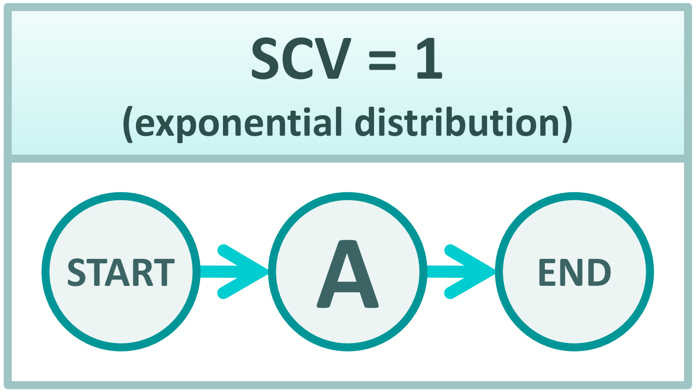

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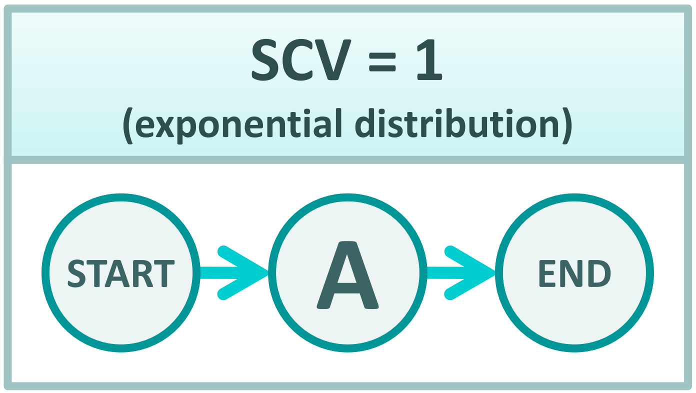
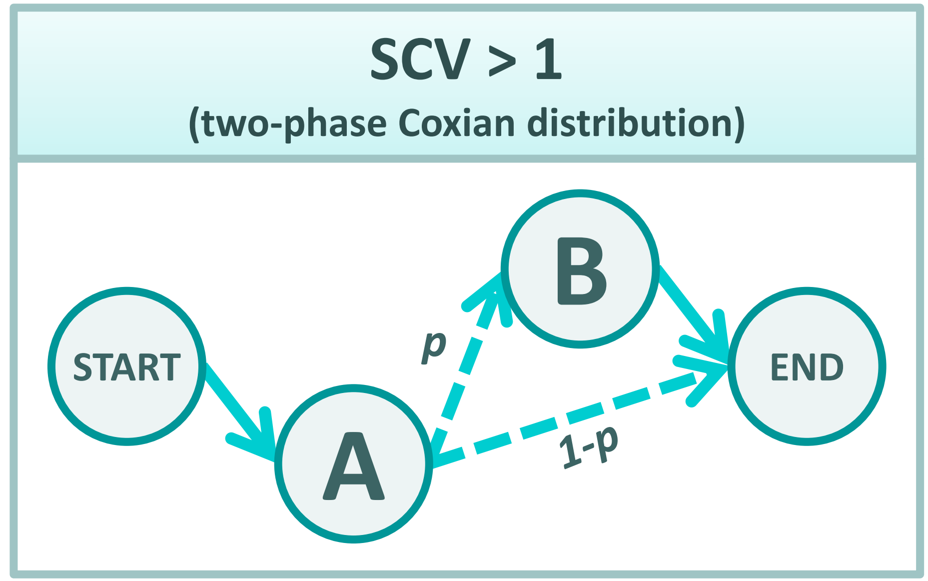
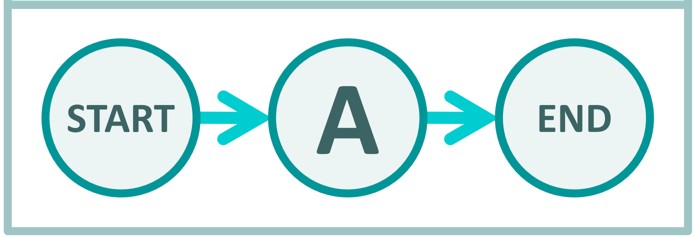


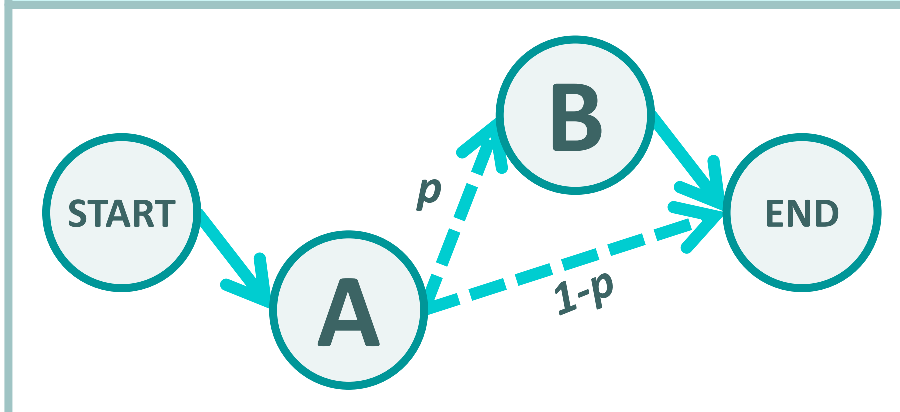


PH distributions: Example of a single activity

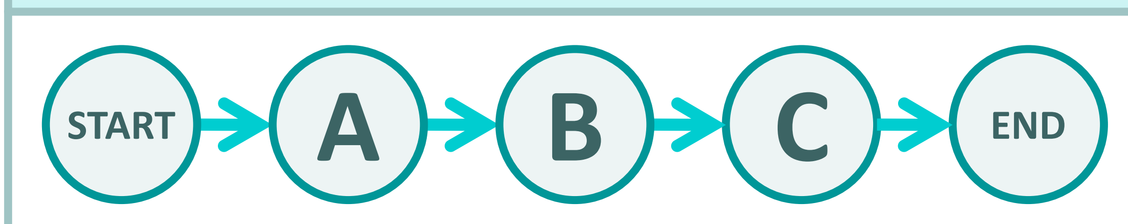
SCV = 1
(exponential distribution)



SCV > 1
(two-phase Coxian distribution)



SCV in [0.333 ; 0.5)
(hypo-exponential distribution)



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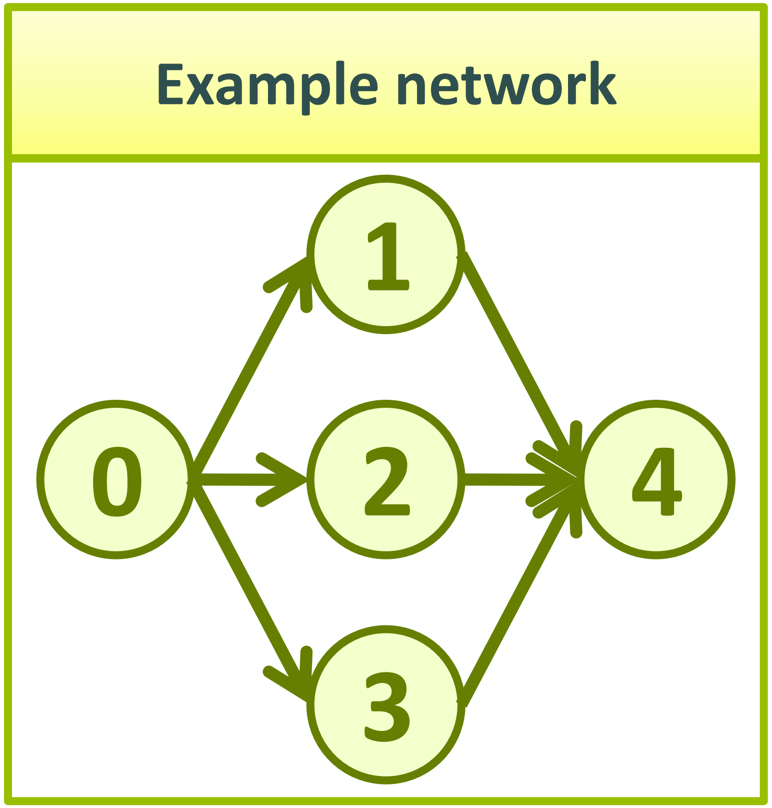
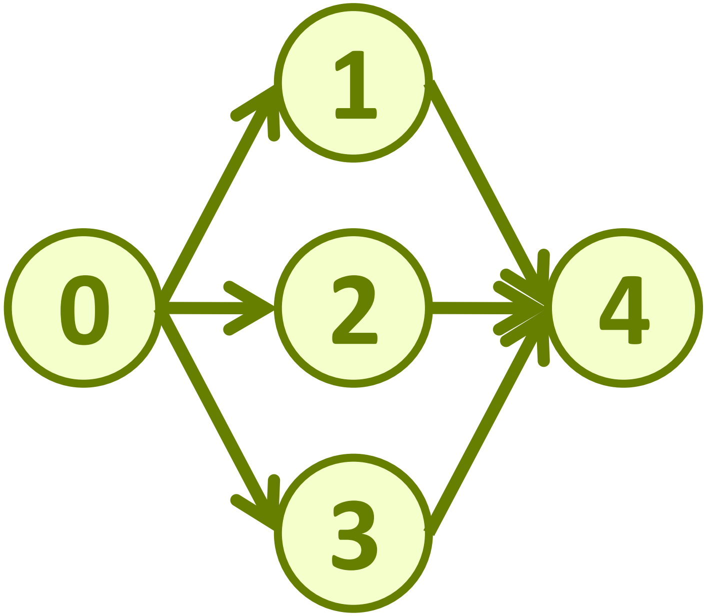


PH distributions: Example of a project network

Activity	SCV
0	Dummy start
1	SCV in [0.33;0.5)
2	SCV = 1
3	SCV > 1
4	Dummy finish

Example network



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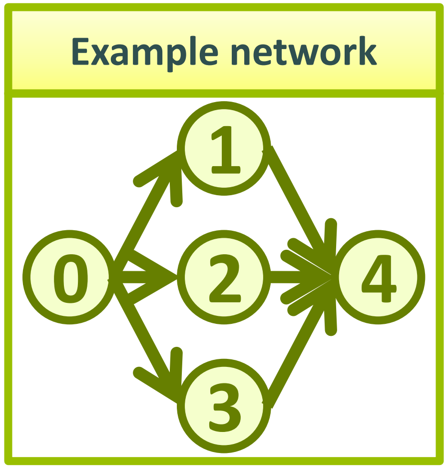

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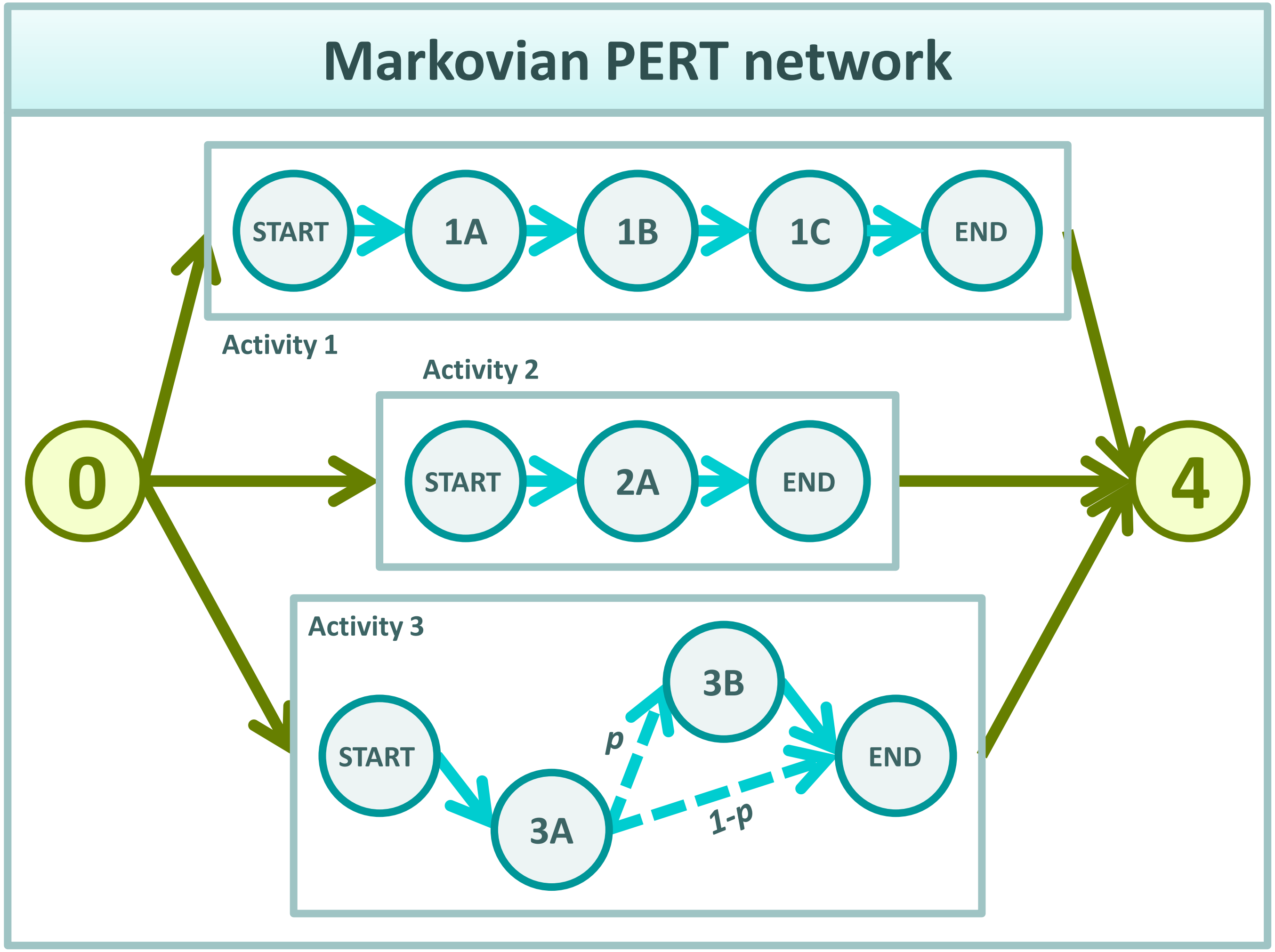

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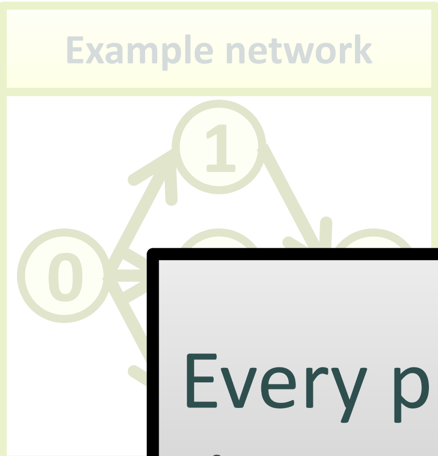

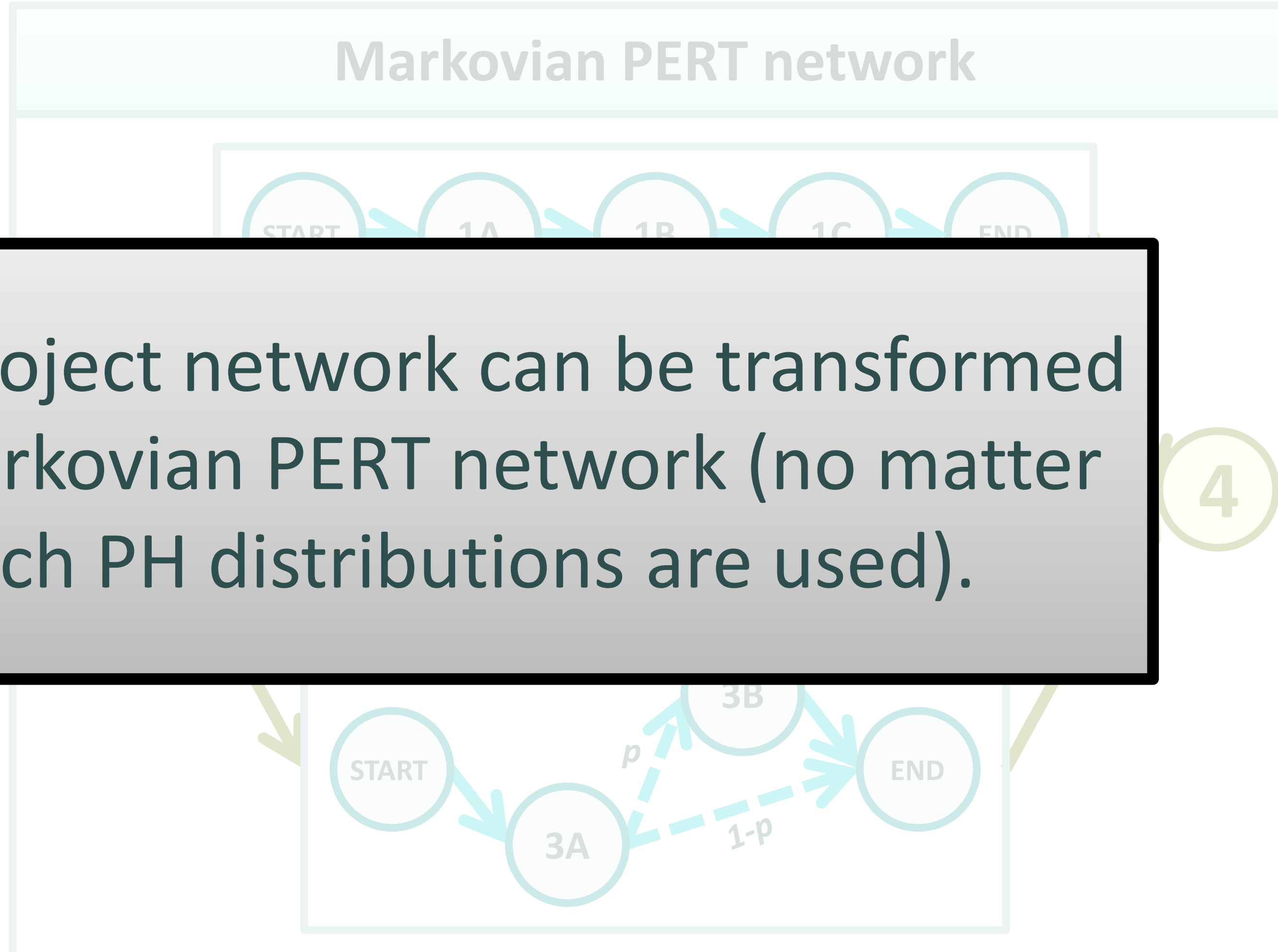

Every project network can be transformed in a Markovian PERT network (no matter which PH distributions are used).

Activity	
0	
1	SCV in [0.33;0.5)
2	SCV = 1
3	SCV > 1
4	Dummy finish

PH distributions: What about low variability?

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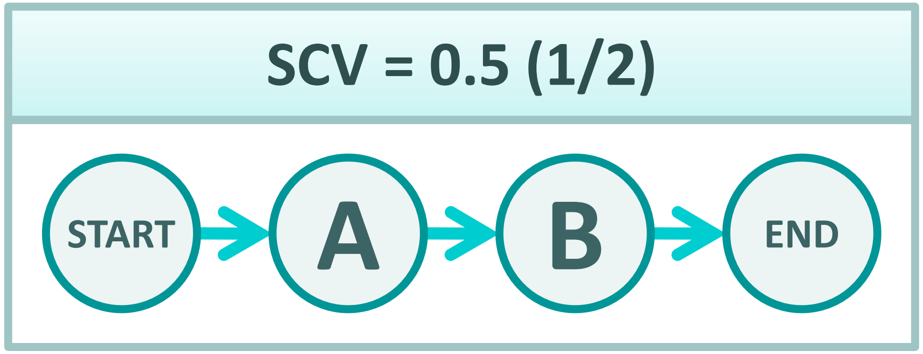

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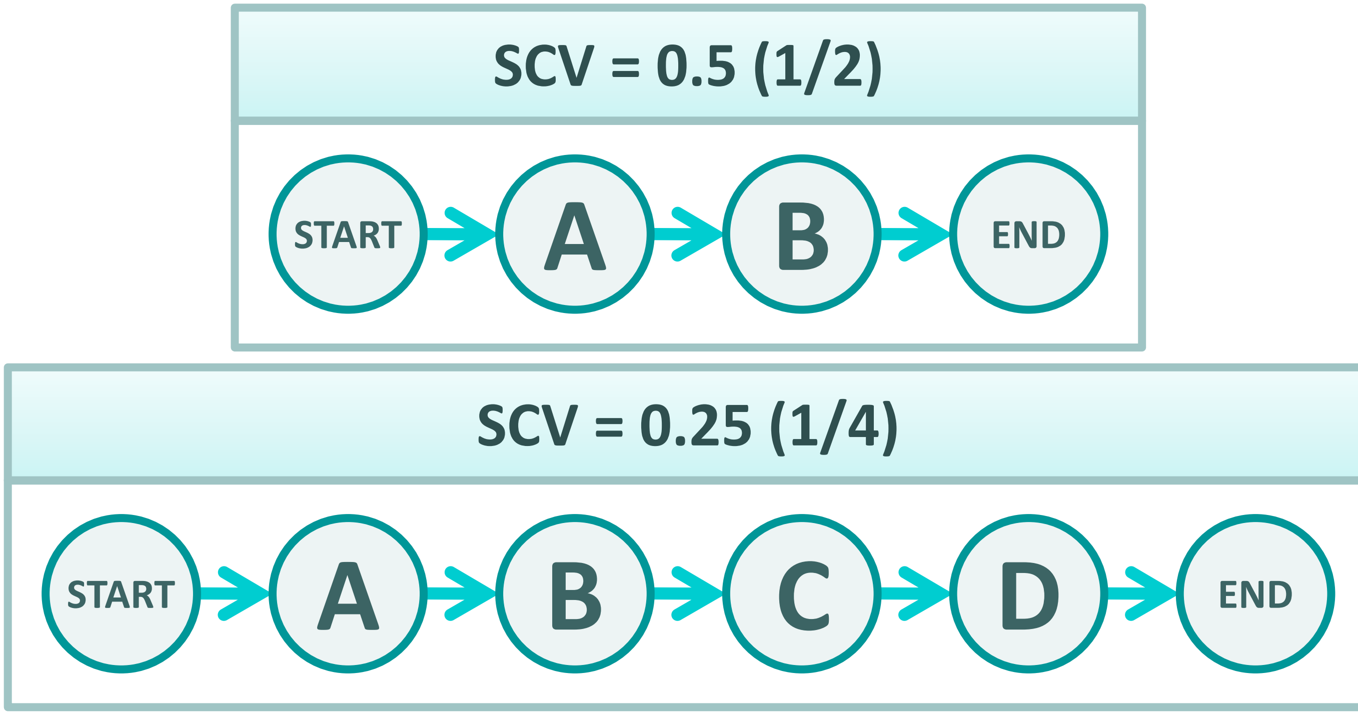

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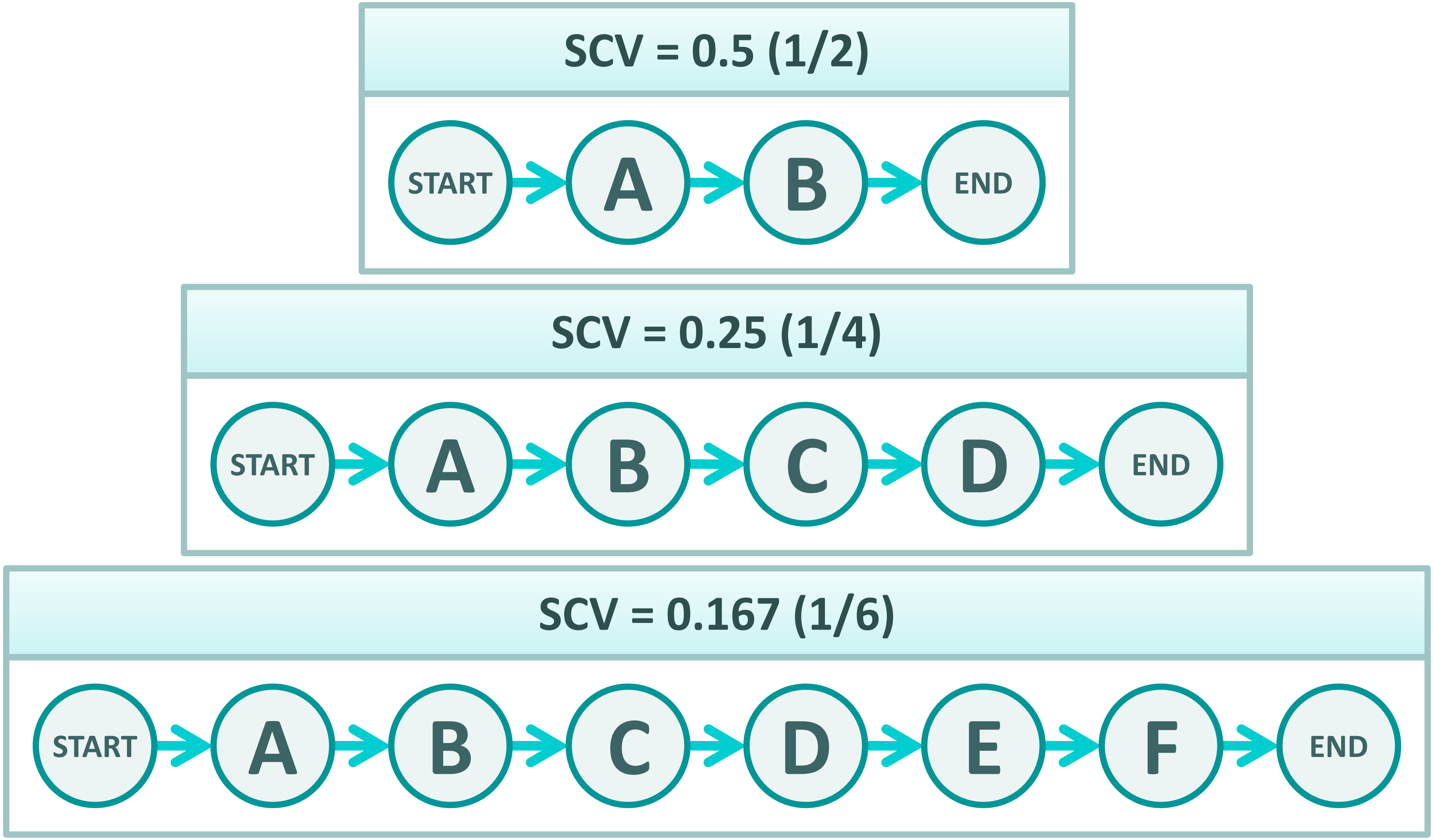

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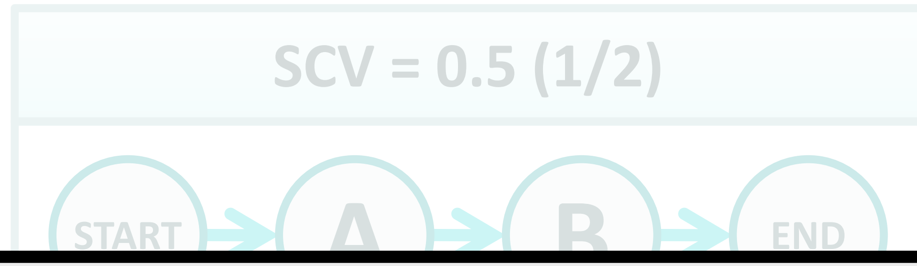

Low variability duration variability inflates the size of the Markovian PERT network.

=>

Our model works best when duration variability is moderate to high.

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Date	1998
Method	Simulated annealing & tabu search
Policy class	RB (Resource-Based)

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Data set					
J30 (PSPLIB)					
J60 (PSPLIB)					
J120 (PSPLIB)					
Patterson					
Golenko					

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J120 (PSPLIB)					**IMPOSSIBLE**
Patterson					
Golenko					**SCV = 0.014**

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  - J60 average improvement of solution quality of 15,9%

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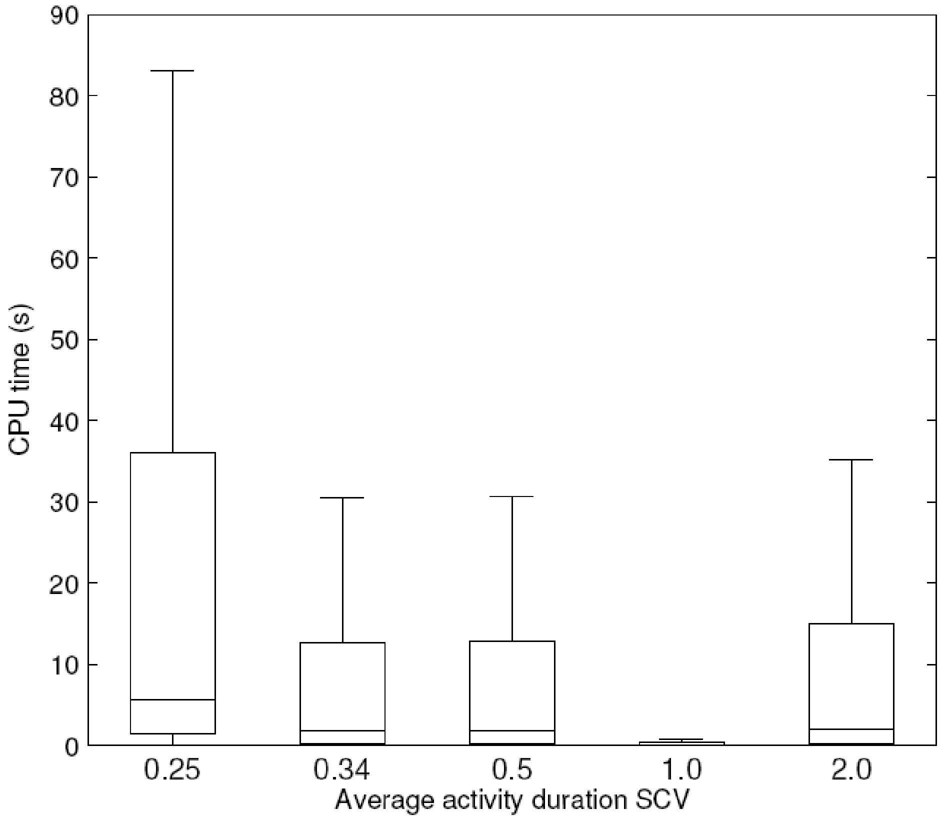

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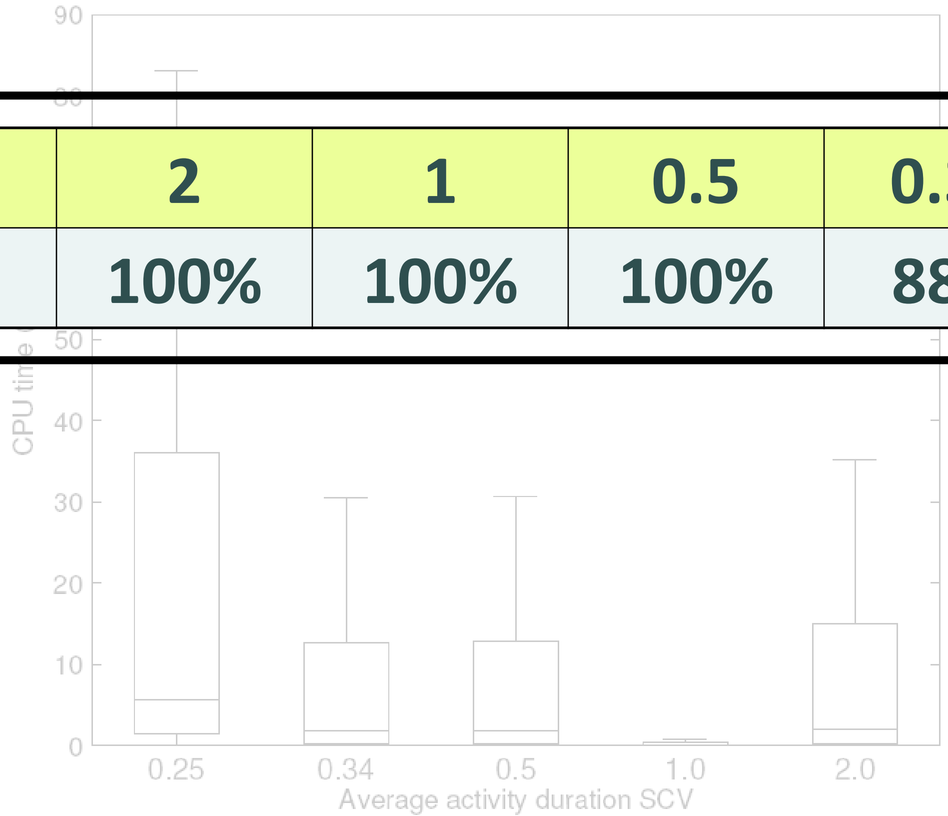
SCV	2	1	0.5	0.34	0.25
% Solved	100%	100%	100%	88%	75%



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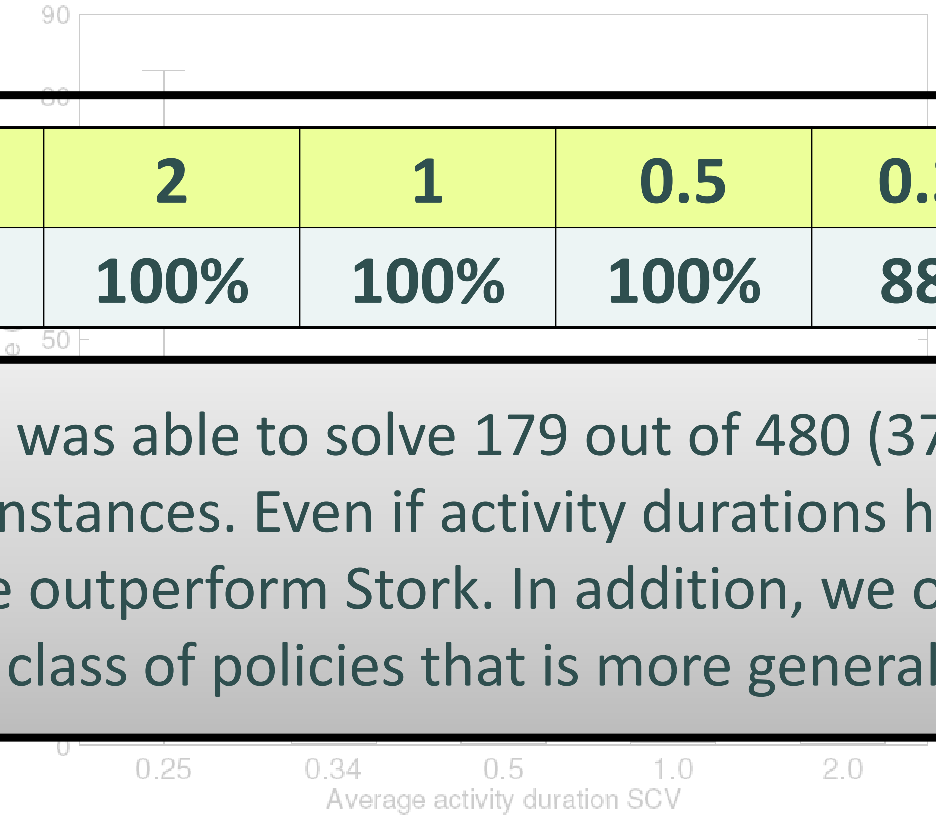
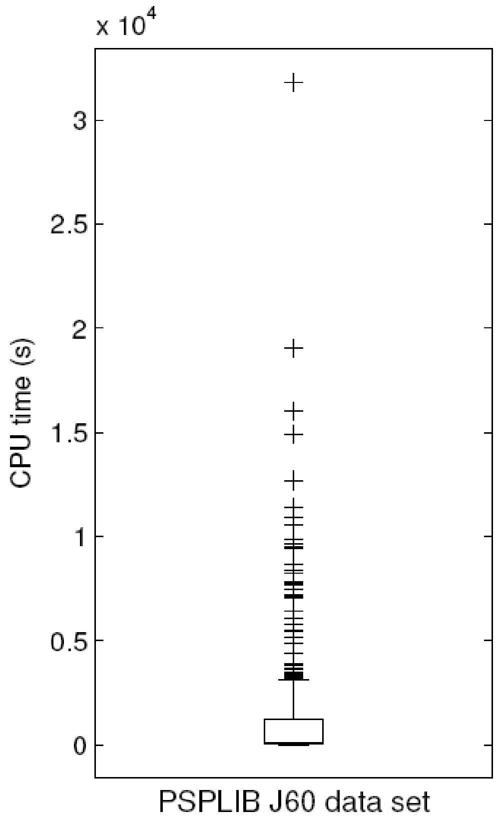


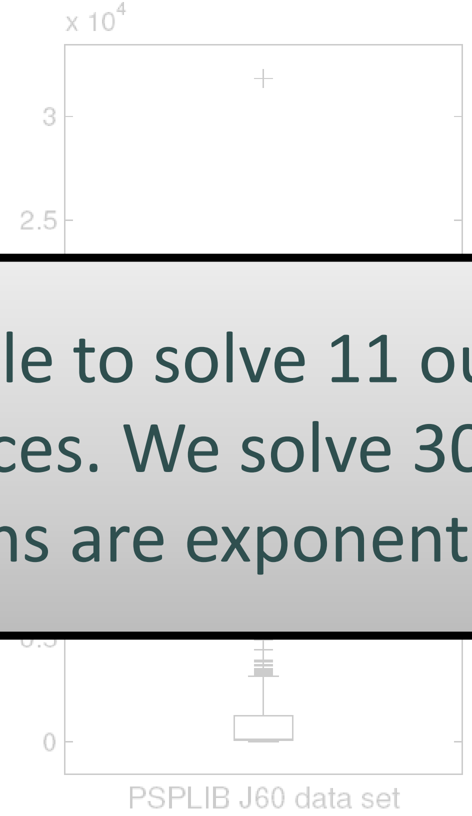
SCV	2	1	0.5	0.34	0.25
% Solved	100%	100%	100%	88%	75%

Stork (2001) was able to solve 179 out of 480 (37%) of the J30 problem instances. Even if activity durations have limited variability, we outperform Stork. In addition, we optimize over a class of policies that is more general!

Results: Computational performance (J60 - PSPLIB)



Results: Computational performance (J60 - PSPLIB)



Stork (2001) was able to solve 11 out of 480 (2%) of the J60 problem instances. We solve 301 instances (63%) if activity durations are exponentially distributed.

Contributions

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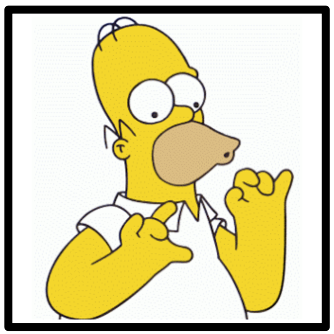

We extend the model of Creemers et al. (2010) in order to solve the SRCPSP. We add resource constraints, general activity durations, and use a minimum-makespan objective.

Solving the SRCPSP makes sense if activities have moderate- to high levels of duration variability. For this setting, our model outperforms the state-of-the art (both in solution quality & in computation speed).

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