Patient Flow Times in the Presence of Outages
A Case Study in a Belgian Hospital

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Problem setting

- Problem setting: healthcare and other services
- Measures of interest:
  - Patient waiting time
- Methodology: queueing theory
  - Focus on manufacturing
  - Healthcare modeling requires distinct approach
A basic queueing system

Arriving patients

Queue

Server

Exit
A basic queueing system

Arrival process

Arriving patients

Queue

Server

Exit
A basic queueing system

Arriving patients: 4, 3, 2, 1

Queue

Service process

Server

Exit
A basic queueing system
A basic queueing system
A basic queueing system

Arriving patients

Queue

Currently in service

Server

Exit
A basic queueing system
A basic queueing system

Arriving patients → Queue → Currently in service → Server → Exit

1

3 2

4
A basic queueing system
Problems in healthcare modeling

- Queue discipline
- Time varying demand
- Waiting creates additional work
- Service outages (absences and interrupts)
- Service epochs
- Reentry at previous workstations
- Probabilistic routing of patients
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Outline

- Problem setting: orthopaedic department of the Middelheim hospital in Antwerp
- Problem: excessive waiting times and congested waiting list
- Objective: analysis of impact of service outages through capacity and variability analysis
- Methodology: queueing models of the orthopaedic department
- Contribution: development of new expressions to assess the impact of service outages
Capacity structure

INFORMS Annual Meeting
Patient Flow Times in the Presence of Outages
Patient flow
Service outages

Different types of outages of the service process:

- Nonpreemptive outages (absences)
- Preemptive outages (interrupts)
- Service epochs (server unavailability)

We formulate queueing models taking these outages into account.
Nonpreemptive outages

- Interruption of the service process prior to treatment of a patient
- Examples:
  - Absence of medical staff at the beginning of a working shift
  - Setup time of medical facilities (e.g. cleaning, preparation)
- Exact results have been obtained in Hopp and Spearman (2000) under the assumption of a fixed number of patients in between two subsequent outages
Nonpreemptive outages: example

Patient 1 | Patient 2 | Patient 3 | Lunch | Patient 4 | Patient 5 | Patient 6
Nonpreemptive outages: example
Preemptive outages

- Interruption of the service process during service itself
- Examples:
  - Emergencies
  - Phone calls, administration, …
- Exact results have been obtained in Hopp and Spearman (2000) under the assumption of:
  - Exponential time between interrupts
  - Interrupts only occur during the service process itself
Preemptive outages

• Example:

| Patient 1 | Patient 2 | Patient 3 | Patient 4 | Patient 5 | Patient 6 |

Exact formulation of mean and variance of service times including preemptive outages:

\[
\mu = \mu_0 \left( \frac{MTTI}{MTTI + MTTR} \right)
\]

\[
\sigma^2 = \sigma_0^2 \left( \frac{MTTI}{MTTI + MTTR} \right)^2 + \frac{\mu_0}{\sigma_r + \left( \frac{MTTR}{MTTI} \right)^2}
\]
Preemptive outages

• Example:

| Patient 1 | Patient 2 | Patient 3 | Patient 4 | Patient 5 | Patient 6 |

\[
\begin{align*}
\mu &= \mu_0 \left( \frac{MTTI}{MTTI + MTTR} \right) \\
\sigma^2 &= \sigma_0^2 \left( \frac{MTTI}{MTTI + MTTR} \right)^2 + \mu_0 \left( \sigma_r^2 + \frac{MTTR^2}{MTTI} \right)
\end{align*}
\]
Preemptive outages

- Example:

```
<table>
<thead>
<tr>
<th>Patient 1</th>
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Preemptive outages

- Example:
Preemptive outages

- Example:

![Patient Flow Times](image)

- Exact formulation of mean and variance of service times including preemptive outages:

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\frac{1}{\mu} = \frac{1}{\mu_0} \left( \frac{MTTI}{MTTI + MTTR} \right)
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\[
\sigma^2 = \sigma_0^2 \left( \frac{MTTI + MTTR}{MTTI} \right)^2 + \frac{1}{\mu_0} \left( \frac{\sigma_r^2 + MTTR^2}{MTTI} \right)
\]
Preemptive outages: generalization

- In healthcare, services may be interrupted during the resolving of a previous interrupt
- Examples:
  - A doctor receiving a phone call during an emergency
  - A doctor who is interrupted by a nurse during a phone call
- We generalize the result of Hopp and Spearman (2000) to include multiple order interrupts
Preemptive outages: generalization

- Example:

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Exact formulation of mean and variance of service times including preemptive outages:

\[
\mu = \mu_0 (MTTI - MTTR) \\
\sigma^2 = MTTI^2 \sigma_0^2 + \mu_0 (MTTI - MTTR)(\sigma_r^2 + MTTR^2)(MTTI - MTTR)^2
\]
Preemptive outages: generalization

**Example:**

![Diagram of patient flow times]

\[ \mu = \mu_0 (MTTI - MTTR) \]

\[ \sigma^2 = MTTI^2 \sigma_0^2 + \mu_0 (MTTI - MTTR) \left( \sigma_r^2 + MTTR^2 \right) (MTTI - MTTR)^2 \]
Preemptive outages: generalization

- Example:

![Diagram of patient flow times with preemptive outages]

\[
\mu = \mu_0 (MTTI - MTTR)
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\[
\sigma^2 = MTTI^2 \sigma_0^2 + \mu_0 (MTTI - MTTR) \left( \sigma_r^2 + MTTR^2 \right) (MTTI - MTTR)
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Preemptive outages: generalization

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\frac{1}{\mu} = \frac{1}{\mu_0} \left( \frac{MTTI}{MTTI - MTTR} \right)
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\sigma^2 = \frac{MTTI^2 \sigma_0^2 + \frac{1}{\mu_0} (MTTI - MTTR) (\sigma_r^2 + MTTR^2)}{(MTTI - MTTR)^2}
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Service epochs

- (Healthcare) services take place during predefined time intervals
- Problem: how to combine surgery, consultation and recovery into one model, while all operate on different time scales:
  - Consultation and surgery take place at weekdays during specific hours
  - Recovery is a continuous process
- Solution: rescaling of the service process using an availability concept
Availability

- Rescales the service process in order to fit a predefined uniform time scale (e.g. 24 hours per day, 7 days per week)
- Example: doctor’s office with opening hours on Thursday from 6 PM until 8 PM and on Friday from 2 PM until 6 PM
- Availability:

\[ A = \frac{6}{168} = \frac{1}{28} \]
Availability

- Rescales the service process in order to fit a predefined uniform time scale (e.g. 24 hours per day, 7 days per week)
- Example: doctor’s office with opening hours on Thursday from 6 PM until 8 PM and on Fridays from 2 PM until 6 PM
- Mean and variance of the rescaled service times:

\[
\frac{1}{\mu} = \frac{1}{A\mu_0}
\]

\[
\sigma^2 = \frac{\sigma_0^2}{A^2}
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- Example: doctor’s office with opening hours on Thursday from 6 PM until 8 PM and on Fridays from 2 PM until 6 PM
- Observe a single week, five arrivals with an average service time of 1 hour
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Methodology

In order to assess patient waiting times at the orthopaedic department, we used a variety of queueing models:

- Parametric decomposition approach
  - Kingman equation: closed form
  - Whitt’s procedure: algorithm
- Brownian queueing model: heavy traffic setting
- Simulation was used as a validation tool

These models were used to test a variety of scenarios, assessing different levels of impact of service outages (absences and interrupts)
Simulation model

Quick facts:

- 60 modules, 18 classes of patients, different phases of treatment
- Single run simulation for each of the scenarios tested
- Number of patients observed each run: 285,000,000 at surgery, 1,150,000,000 at consultation
- Simulation runtime: 86,000 years
- Resulting statistical precision: standard error < 0.00001
### Results: base model

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{\mu_i}$</td>
<td>0.01257</td>
<td>0.06329</td>
<td>0.79710</td>
<td>5.03237</td>
<td>8.09661</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.99543</td>
<td>0.97854</td>
<td>0.14776</td>
<td>0.75701</td>
<td>0.20396</td>
</tr>
<tr>
<td>$C^2_{s_i}$</td>
<td>0.65079</td>
<td>0.60612</td>
<td>14.0786</td>
<td>1.98721</td>
<td>23.4125</td>
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<tr>
<td>$E[ W_{\text{Kingman}}]$</td>
<td>5.05894</td>
<td>3.95430</td>
<td>0.79710</td>
<td>5.24027</td>
<td>8.09687</td>
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<tr>
<td>$E[ W_{\text{Whitt}}]$</td>
<td>5.05911</td>
<td>3.95298</td>
<td>0.79710</td>
<td>5.20325</td>
<td>8.09664</td>
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<tr>
<td>$E[ W_{\text{Brownian}}]$</td>
<td>7.72261</td>
<td>5.41723</td>
<td>0.27924</td>
<td>1.19658</td>
<td>5.00118</td>
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<td>$\rho_i$</td>
<td>0.99541</td>
<td>0.97858</td>
<td>0.14775</td>
<td>0.75701</td>
<td>0.20414</td>
</tr>
<tr>
<td>$C^2_{s_i}$</td>
<td>0.65796</td>
<td>0.60589</td>
<td>14.0969</td>
<td>1.98918</td>
<td>23.9050</td>
</tr>
<tr>
<td>$E[ W_{\text{Simulation}}]$</td>
<td>5.40098</td>
<td>3.46204</td>
<td>0.79711</td>
<td>5.11928</td>
<td>8.10131</td>
</tr>
</tbody>
</table>
Results: scenarios

Patient waiting time (days)

- **Brownian**
- **Simulation**
- **Kingman**
- **Whitt**
conclusions

- When assessing waiting time at complex hospital systems, parametric decomposition approaches work best
- Hospital decision makers should avoid the high utilization trap
- Decreasing the size, amount and variability of service outages is able to yield significant improvement

Contributions:
- Development of new expressions to model service outages
- Comparison of different modeling techniques
Time for questions