A new algorithm to optimize a can-order inventory policy for two companies in a horizontal partnership

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Agenda

• Horizontal cooperation: what, why, how?
• Examples of horizontal cooperations
• Definitions & assumptions
• Problem Setting Example
• Costs & Performance Measures
• Methodology
• Numerical Example
• Future research
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Horizontal Cooperation

• **What** = cooperation where companies bundle their orders/join shipments
Horizontal Cooperation

- **What** = cooperation where companies bundle their orders/join shipments
- **Why** = to reduce transport costs, CO2 emissions, and congestion
Horizontal Cooperation

- **What** = cooperation where companies bundle their orders/join shipments
- **Why** = to reduce transport costs, CO2 emissions, and congestion
- **How** = by using the available space in truck hauls of one company to ship items of another company
Horizontal Cooperation

- **What** = cooperation where companies bundle their orders/join shipments
- **Why** = to reduce transport costs, CO2 emissions, and congestion
- **How** = by using the available space in truck hauls of one company to ship items of another company
- **Vertical cooperation** = cooperation with companies at different level of the supply chain (e.g., supplier & buyers)
- **Horizontal cooperation** = cooperation with companies at the same level of the supply chain
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Examples of Horizontal Cooperation

Tupperware

P&G
Examples of Horizontal Cooperation

- Tupperware
- P&G
- Nestlé
- Pepsi
Examples of Horizontal Cooperation

- Tupperware
- P&G
- Nestlé
- pepsi
- ucb
- Baxter
Examples of Horizontal Cooperation

What do we observe?
1. Horizontal cooperations can be established even with competitors!
2. Horizontal cooperations often only have 2 partners.
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 Definitions & Assumptions

• Assumptions:
  – Two companies
  – Both companies adopt a $\text{(S,c,s)}$ can-order policy to synchronize their orders
  – No replenishment lead time
  – Unit Poisson demand (iid for both companies)

• Definitions:
  – $I_i$: the inventory level at company $i$
  – $S_i$: the order-up-to level of company $i$
  – $c_i$: the can-order level of company $i$
  – $s_i$: the reorder point of company $i$
  – $l_i$: the Poisson arrival rate of customers at company $i$
Definitions & Assumptions

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Problem Setting Example \((t = t_0)\)

Company 1

\[ S_1 \]

\[ C_1 \]

\[ l_1 \]

Company 2

\[ S_2 \]

\[ C_2 \]

\[ l_2 \]
Problem Setting Example \((t = t_1)\)
Problem Setting Example \((t = t_1)\)
Problem Setting Example \((t = t_2)\)
Problem Setting Example \( (t = t_2) \)
Problem Setting Example \((t = t_3)\)

Company 1

\[ S_1 \]

\[ c_1 \]

\[ S_1 \]

\[ i_1 \]

Company 2

\[ S_2 \]

\[ c_2 \]

\[ S_2 \]

\[ i_2 \]
Problem Setting Example \((t = t_3)\)

- **Company 1**
  - \(S_1\)
  - \(c_1\)
  - \(I_1\)

- **Company 2**
  - \(S_2\)
  - \(c_2\)
  - \(I_2\)
Problem Setting Example \((t = t_3)\)
Problem Setting Example \((t = t_3)\)
Problem Setting Example ($t = t_3$)
Problem Setting Example \((t = t_x)\)
Problem Setting Example \((t = t_{x+1})\)
Problem Setting Example \((t = t_{x+1})\)

Company 1

\[
\begin{align*}
S_1 \\
C_1 \\
S_1 \\
I_1
\end{align*}
\]

Company 2

\[
\begin{align*}
S_2 \\
C_2 \\
S_2 \\
I_2
\end{align*}
\]
Problem Setting Example \((t = t_{x+1})\)

Company 1

\[ S_1 \]

\[ c_1 \]

\[ S_1 \quad \text{Company 1} \]

\[ I_1 \]

Company 2

\[ S_2 \]

\[ c_2 \]

\[ S_2 \quad \text{Company 2} \]

\[ I_2 \]

Example \((t = t_{x+1})\)
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• **Costs & Performance Measures**
• Methodology
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Costs & Performance Measures

• Costs:
  – $K$ = major order cost
  – $k_i$ = the minor order cost for company $i$
  – $h_i$ = the unit holding cost for company $i$
Costs & Performance Measures

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Costs & Performance Measures

• Costs:
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  – $k_i$ = the minor order cost for company $i$
  – $h_i$ = the unit holding cost for company $i$

• Performance measures of interest:
  – The number of times company $i$ orders first ($K$ & $k_i$ are incurred)
  – The number of times company $i$ joins the order of company $j$ (only $k_i$ is incurred)
Costs & Performance Measures

• Costs:
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  \( \Rightarrow \) The order cost for both companies
Costs & Performance Measures

• Costs:
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  ⇒ The order cost for both companies
  – The average inventory at company \( i \)
Costs & Performance Measures

• Costs:
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• Performance measures of interest:
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  ⇒ The order cost for both companies
  – The average inventory at company $i$
  ⇒ The inventory holding cost for both companies
Costs & Performance Measures

• Costs:
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  – $k_i =$ the minor order cost for company $i$
  – $h_i =$ the unit holding cost for company $i$

• Performance measures of interest:
  – The number of times company $i$ orders first ($K$ & $k_i$ are incurred)
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    (only $k_i$ is incurred)
  ⇒ The order cost for both companies
  – The average inventory at company $i$
  ⇒ The inventory holding cost for both companies
  ⇒ The total cost for both company given their $(S,c,s)$ policy
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Methodology

• **Goal** = to find $\Pi$, the optimal $(S,c,s)$ can-order policy for both companies
Methodology

• **Goal** = to find $\Pi$, the optimal $(S,c,s)$ can-order policy for both companies

• **How:**
  1. Evaluate the performance of a single $(S,c,s)$ policy
  2. Enumerate all policies in order to find the optimal policy
Methodology

- **Goal** = to find $\Pi$, the optimal $(S,c,s)$ can-order policy for both companies

- **How:**
  1. Evaluate the performance of a single $(S,c,s)$ policy
  2. Enumerate all policies in order to find the optimal policy

- **Available methodologies:**
  - Simulation
  - Markov chains
  - A new approach?
Methodology

• Simulation = too time-consuming
Methodology

- Simulation = too time-consuming
- Markov chains:
  - State space can be represented by double \((I_1, I_2)\)
  - State-space size is \((S_1 \times S_2)\)
  - For \(S_1 = S_2 = 1,000\), the number of states equals 1,000,000
Methodology

• Simulation = too time-consuming
• Markov chains:
  – State space can be represented by double \((I_1, I_2)\)
  – State-space size is \((S_1 \times S_2)\)
  – For \(S_1 = S_2 = 1,000\), the number of states equals 1,000,000
    \(\Rightarrow\) Markov chains cannot be used for real-life problems
Methodology

• Simulation = too time-consuming
• Markov chains:
  – State space can be represented by double \((l_1, l_2)\)
  – State-space size is \((S_1 \times S_2)\)
  – For \(S_1 = S_2 = 1,000\), the number of states equals 1,000,000
    \(\Rightarrow\) Markov chains cannot be used for real-life problems
  – In addition, it is difficult to obtain the number of (first/joined) orders using Markov chains
Methodology

• Simulation = too time-consuming

• Markov chains:
  – State space can be represented by double \((I_1, I_2)\)
  – State-space size is \((S_1 \times S_2)\)
  – For \(S_1 = S_2 = 1,000\), the number of states equals 1,000,000
    \(\Rightarrow\) Markov chains cannot be used for real-life problems
  – In addition, it is difficult to obtain the number of (first/joined) orders using Markov chains

• A new approach:
  – Also uses a Markov chain
  – State-space size is at most \((S_1 + S_2)\)
  – For \(S_1 = S_2 = 1,000\), the number of states equals at most 2,000
Methodology

• Simulation = too time-consuming

• Markov chains:
  – State space can be represented by double $(l_1, l_2)$
  – State-space size is $(S_1 \times S_2)$
  – For $S_1 = S_2 = 1,000$, the number of states equals 1,000,000
    \[\Rightarrow\] Markov chains cannot be used for real-life problems
  – In addition, it is difficult to obtain the number of (first/joined) orders using Markov chains

• A new approach:
  – Also uses a Markov chain
  – State-space size is at most $(S_1 + S_2)$
  – For $S_1 = S_2 = 1,000$, the number of states equals at most 2,000
    \[\Rightarrow\] 500 times smaller!
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<td>2</td>
<td>2</td>
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<td>$s_i$</td>
<td>0</td>
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<tr>
<td>$\lambda_i$</td>
<td>1</td>
<td>9</td>
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Assume we start from a “full” system
• There is a 10% probability that the next customer visits company 1
• There is a 90% probability that the next customer visits company 2
Law of competing exponentials

- There is a 10% probability that the next customer visits company 1
- There is a 90% probability that the next customer visits company 2
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**Law of competing exponentials**

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \frac{\lambda_2}{\lambda_1 + \lambda_2}
\]
Law of competing exponentials

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2}
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<th>C_i</th>
<th>S_j</th>
<th>λ_j</th>
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Law of competing exponentials

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \text{and} \quad \frac{\lambda_2}{\lambda_1 + \lambda_2}
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**Law of competing exponentials**

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \text{for } x
\]

\[
\frac{\lambda_2}{\lambda_1 + \lambda_2} \quad \text{for } y
\]
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**Law of competing exponentials**

$$x \sim \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

$$y \sim \frac{\lambda_2}{\lambda_1 + \lambda_2}$$
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**Law of competing exponentials**

$$\frac{\lambda_1}{\lambda_1 + \lambda_2}$$

$$\frac{\lambda_2}{\lambda_1 + \lambda_2}$$
Law of competing exponentials

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \frac{\lambda_2}{\lambda_1 + \lambda_2}
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Law of competing exponentials

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**Law of competing exponentials**

\[
x \sim \frac{\lambda_1}{\lambda_1 + \lambda_2}
\]

\[
y \sim \frac{\lambda_2}{\lambda_1 + \lambda_2}
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Law of competing exponentials

\[
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Law of competing exponentials

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Law of competing exponentials

\[ \frac{\lambda_1}{\lambda_1 + \lambda_2} \]
Law of competing exponentials

\[ \frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_1 + \lambda_2} \]
Law of competing exponentials

\[
\frac{x}{y} = \frac{\lambda_1}{\lambda_1 + \lambda_2}
\]

\[
\frac{\lambda_2}{\lambda_1 + \lambda_2}
\]
Law of competing exponentials

\[
\frac{x}{\lambda_1 + \lambda_2} = \frac{\lambda_1}{\lambda_1 + \lambda_2}
\]

\[
\frac{y}{\lambda_1 + \lambda_2} = \frac{\lambda_2}{\lambda_1 + \lambda_2}
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Law of competing exponentials

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\frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \frac{\lambda_2}{\lambda_1 + \lambda_2}
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Law of competing exponentials

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\frac{x}{\lambda_1 + \lambda_2} = \frac{\lambda_2}{\lambda_1 + \lambda_2}
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Law of competing exponentials

\[
x = \frac{\lambda_1}{\lambda_1 + \lambda_2}
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\[
y = \frac{\lambda_2}{\lambda_1 + \lambda_2}
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Law of competing exponentials

\[
\begin{align*}
\frac{\lambda_1}{\lambda_1 + \lambda_2} & \quad x \\
\frac{\lambda_2}{\lambda_1 + \lambda_2} & \quad y
\end{align*}
\]
Law of competing exponentials

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2}
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\[ \frac{\lambda_1}{\lambda_1 + \lambda_2} \]

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Law of competing exponentials

\[
\begin{align*}
\frac{\lambda_1}{\lambda_1 + \lambda_2} & \quad x \leftarrow y \\
\frac{\lambda_2}{\lambda_1 + \lambda_2} &
\end{align*}
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Law of competing exponentials

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**Law of competing exponentials**

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x = \frac{\lambda_1}{\lambda_1 + \lambda_2}
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y = \frac{\lambda_2}{\lambda_1 + \lambda_2}
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Law of competing exponentials

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Law of competing exponentials

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Regular states (visit probability obtained using binomial distribution)
Law of competing exponentials

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2}
\]

\[
\frac{\lambda_2}{\lambda_1 + \lambda_2}
\]

**Final states** (visit probability obtained using negative binomial distribution)
Law of competing exponentials

\[
\frac{x}{\lambda_1 + \lambda_2} = \frac{\lambda_1}{\lambda_1 + \lambda_2}
\]

Initial states (visit probability obtained using negative binomial distribution)
Law of competing exponentials

\[
\begin{align*}
\frac{\lambda_1}{\lambda_1 + \lambda_2} &
\end{align*}
\]

Initial states (visit probability obtained using negative binomial distribution)
Law of competing exponentials

\[
\begin{align*}
\frac{\lambda_1}{\lambda_1 + \lambda_2} & \quad \text{Initial states (visit probability obtained using negative binomial distribution)} \\
\frac{\lambda_2}{\lambda_1 + \lambda_2} &
\end{align*}
\]
We have a Markov chain that holds the probabilities to move from one initial state towards another.
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From this Markov chain, we can obtain the steady-state probabilities to visit one of the initial states!
We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state.
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Recall that the visit probabilities of the regular states can easily be obtained using the binomial distribution.
We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state.

We obtain the steady-state probabilities to visit any of the regular states as the weighted sum of probabilities to visit the regular states when departing from a given initial state.

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We obtain the steady-state probabilities to visit any of the regular states as the weighted sum of probabilities to visit the regular states when departing from a given initial state.

Using the steady-state probabilities to visit the regular states, we can easily calculate the expected inventory at each company.
We can also use these steady-state probabilities to weigh the probability to visit a final state when departing from an initial state.
We can also use these steady-state probabilities to weigh the probability to visit a **final** state when departing from an **initial** state.
We can also use these steady-state probabilities to weigh the probability to visit a final state when departing from an initial state.

Recall that the visit probabilities of the final states can easily be obtained using the negative binomial distribution.
We can also use these steady-state probabilities to weigh the probability to visit a final state when departing from an initial state.

Again, we obtain the steady-state probabilities to visit any of the final states as the weighted sum of probabilities to visit the final states when departing from a given initial state.

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Again, we obtain the steady-state probabilities to visit any of the final states as the weighted sum of probabilities to visit the final states when departing from a given initial state.

Given the number of transitions it takes to move from an initial state to a final state, we can calculate the number of times a company places a single/joined order.

Recall that the visit probabilities of the final states can easily be obtained using the negative binomial distribution.
Numerical Example: Conclusions

• If we use a regular Markov chain to model the example:
  – We end up with 24 states
  – We cannot easily calculate the number of orders (joined/single) for each company

• If we use our new approach:
  – We end up with a Markov chain of 5 states
  – We can easily obtain both inventory holding costs and order costs (i.e., the total cost of the coordination)
Agenda

• Horizontal cooperation: what, why, how?
• Examples of horizontal cooperations
• Definitions & assumptions
• Problem Setting Example
• Costs & Performance Measures
• Methodology
• Numerical Example

• Future research
Future/Current Research

• We can use our model to investigate/compare the costs in a coordination and the standalone costs (cfr. Valeria’s talk next session)

• Because our model is fast/efficient, we can use it to study the characteristics of the optimal policy in a two-company horizontal cooperation

• Lastly, we also relax the assumptions:
  – Non-zero & non-exponential lead times
  – Non-exponential customer interarrival times
  – (S, c, Q) order policy
  – Truck capacity constraints
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